# Circular sectors

## Trigonometry 2

INU0115/515 (Maths 2)

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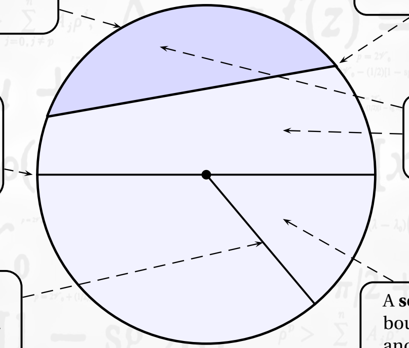
# Circle definitions

The **circumference** is the total distance around the edge of the circle.

A **chord** is a line across the circle but not passing through the centre.



The **diameter** is the length of the straight line passing through the centre of the circle.

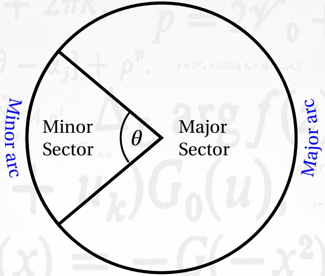
A chord splits the circle into a **major segment** and a **minor segment**.

The **radius** is a straight line from centre to circumference. It is half the diameter.

A **sector** is a region bounded by two radii and an arc.

# Sectors of circles

A circle can be split into sectors by dividing it along two radii.




The largest slice is called the *major* sector and the smaller slice is the *minor* sector.

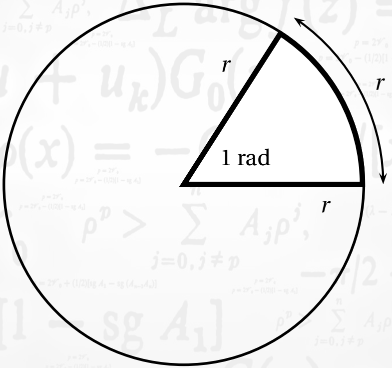
In the picture above we'd say 'the minor sector *subtends* an angle $\theta$ at the centre of the circle'.

The curved edge of the minor sector is called a *minor arc.* The curved edge of the major sector is called a *major arc.*

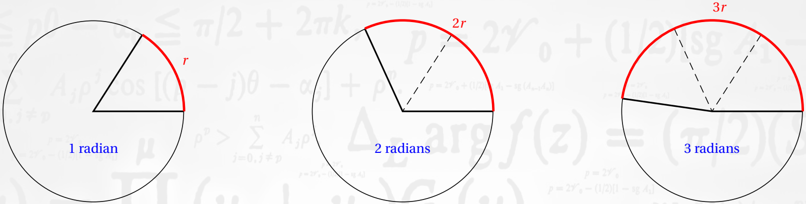
# Definition of a radian

The picture shows a circle of radius $r$ and a rotation which is just enough so that the minor arc traced out is equal in length to the radius.

This angle is defined as 1 *radian* (or 1 rad).



Let's examine the link between radian angle and arc length.




1 rad ⇒ Arc length: $r$. 2 rad ⇒ Arc length: $2r$. 3 rad ⇒ Arc length: $3r$.

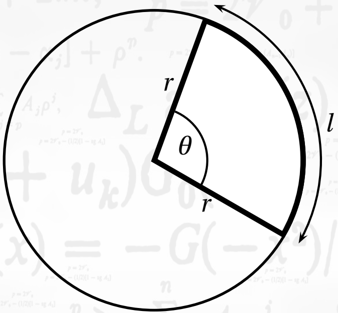
...and so on. An angle of $\theta$ radians will give an arc length of $\theta r$.

In other words...the relationship between arc length and angle is

$$\text{Arc length} = \text{Angle in radians} \times \text{Radius}$$

# Arc length of a sector

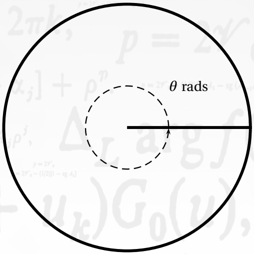
The general case: a circular sector with an arc length $l$ subtending an angle $\theta$ radians at the centre.




Then

$$\begin{aligned} \text{Arc length} &= \text{Radius} \times \text{Angle in radians} \\ \therefore l &= r\theta \end{aligned}$$

# How many radians in a circle?



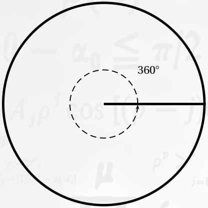

Consider the case where $l$ is equal to the circumference of the circle ($2\pi r$). How many radians is this?

Using $l = r\theta$ then

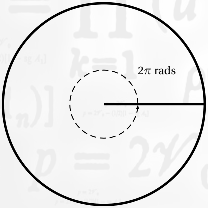
$$2\pi r = r\theta \qquad \Rightarrow \theta = 2\pi \text{ rads}$$

There are $2\pi$ radians ($\approx 6.283$ radians) in a circle.

# Measuring rotation




We can measure angles using the *degree* and is represented by the symbol (°). You should be already familiar with the idea that there are 360° in a full rotation, or circle.




Angles can also be measured using the *radian*, represented by the symbol ($^c$), or by rad. One complete rotation is equivalent to $2\pi$ radians.

Radians are a *natural* measure of rotation. They must be used in many formulae and equations in science and engineering.

# Converting between degrees and radians

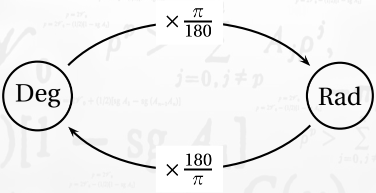
A complete rotation of 360° is equivalent to $2\pi$ radians. Therefore

$$360^\circ = 2\pi\,\text{rad}$$

From this we can obtain the following conversions:

$$1^\circ = \frac{\pi}{180}\ \text{rad} \quad \text{and} \quad 1\ \text{rad} = \frac{180^\circ}{\pi}$$

Pictorially, the procedure for converting is:



## Converting degrees to radians

Express 72° in radians, and in terms of $\pi$.

$$72^\circ = 72 \times \frac{\pi}{180} \text{ rad} = \frac{72\pi}{180} \text{ rad} = \frac{2\pi}{5} \text{ rad}$$

## Converting radians to degrees

Express $\frac{3\pi}{4}$ rad, in degrees.

$$\frac{3\pi}{4} \text{ rad} = \frac{3\pi}{4} \times \frac{180^\circ}{\pi} = \frac{3}{4} \times 180^\circ = 135^\circ$$

## Converting degrees to radians

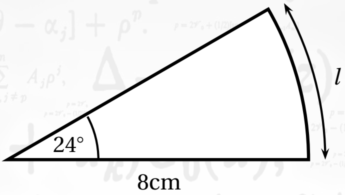
Express 285° in radians, to 5 decimal places.

$$285^\circ = 285 \times \frac{\pi}{180} \text{ rad} = 4.97419 \text{ rad}$$

## Finding the arc length of a sector

A sector of a circle with radius 8cm subtends an angle of 24°. Calculate the exact arc length of the sector.

Here is the picture:




The angle is given in degrees so we convert to radians before using the arc length formula.

$$24^\circ = \frac{24\pi}{180} = \frac{2\pi}{15}\text{rads}$$

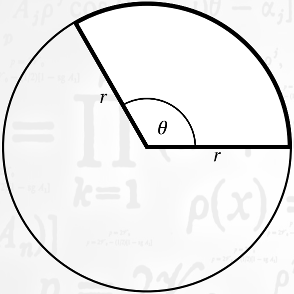
The arc length is given by $l = r\theta$ so

$$l = 8 \times \frac{2\pi}{15} = \frac{16\pi}{15}\text{cm}$$

So the arc length of the sector is approximately 3.35cm.

# Area of a sector

Here is a sector of radius $r$ subtending an angle $\theta$.




In a *full* circle $\theta = 2\pi$.
In *half* a circle $\theta = \pi$.
In a *quarter* circle $\theta = \frac{1}{2}\pi$.

*Sector area is proportional to the angle $\theta$.*

*(e.g. if we half the angle then the sector area will also be halved).*

In fact the sector area depends on the *ratio* of $\theta$ to a full circle of $2\pi$ radians.

$$\begin{aligned} \text{Sector area} &= \frac{\theta}{2\pi} \times \text{Circle area} \\ &= \frac{\theta}{2\pi} \times \pi r^2 \\ \therefore \text{Sector area} &= \tfrac{1}{2} r^2 \theta \end{aligned}$$

Remember: the angle $\theta$ must be given in radians when using this formula.

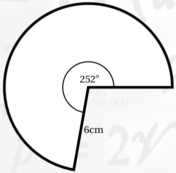
## Definition

The perimeter of a shape is the distance around the outside. For example, the perimeter of a square is the sum of the lengths of the four sides. The perimeter of a circle is the same as its circumference.

## Calculating the area of a sector

A sector of a circle with radius 6cm which subtends an angle of 252° at its centre. Calculate the area and perimeter of the sector, giving the answers to 3 S.F.

Start with a little sketch.




The angle $\theta$ must be in radians:

$$\theta = \frac{252\pi}{180} = \frac{7\pi}{5}\,\text{rads}$$

The area of the sector is:

$$\text{Area} = \tfrac{1}{2} \times 6^2 \times \frac{7\pi}{5} = \frac{126\pi}{5}\,\text{cm}^2$$

So, to 3SF the area is 79.2cm$^2$.

To calculate the *perimeter* we need to find the the distance around the edge of the sector.

First, the arc length:

$$l = 6 \times \frac{7\pi}{5} = \frac{42\pi}{5}\,\text{cm}$$

The perimeter of the sector is $l + 2r$ so, to 3SF it is $6 + 6 + \frac{42\pi}{5} = 38.4$cm.

# Test yourself...

Use your knowledge to answer the following questions.

1. Express 210° in radians.
2. Express $\frac{3\pi}{10}$ radians in degrees
3. A circular sector of radius 8 mm subtends an angle of 60°. Find the arc length and area.
4. A circular sector has an area of 30 $\text{cm}^2$. Given that the radius is 8 cm calculate the angle subtended at the centre.

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Answers:

1. $\frac{7}{6}\pi$ radians
2. 54°
3. Arc length $= \frac{8\pi}{3}$ mm, Area $= \frac{32\pi}{3}$ $\text{mm}^2$
4. Angle $\theta = \frac{15}{16}$ radians.