Trig Functions and Graphs

Trigonometry 1

INU0115/515 (Maths 2)

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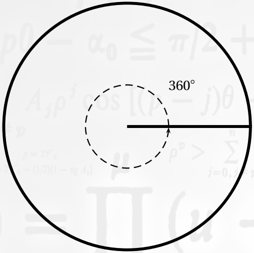
Objectives

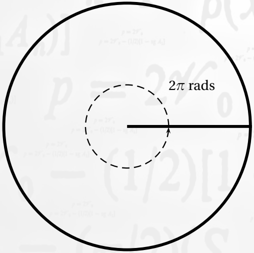
In this presentation we'll meet some basic concepts associated with angles and trig functions.

- Measuring rotation and angles
    - Converting between degrees and radians
    - Positive and negative angles
- Definitions and properties of sine, cosine and tangent functions
- Graphs of the trig functions

Measuring rotation

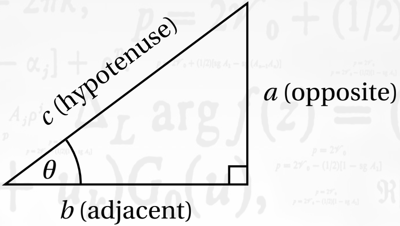


We can measure angles using the *degree* and is represented by the symbol (°).
You should be already familiar with the idea that there are 360° in a full rotation, or circle.



Angles can also be measured using the *radian,* represented by the symbol (^{c}), or by rad. One complete rotation is equivalent to 2π radians.
Radians are a more natural measure of rotation and they must be used in some of the equations which occur in science and engineering.

Trig ratios for a right-angle triangle



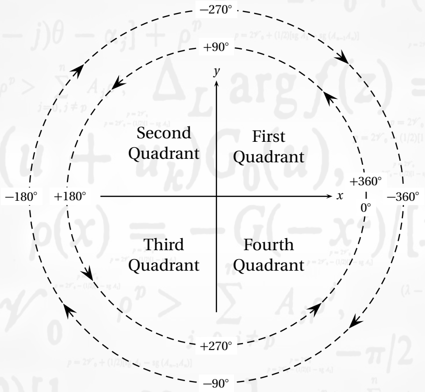
Trigonometric ratios, defined for the angle θ are called sine, cosine and tangent (usually shortened to “sin”, “cos” and “tan” respectively).

With reference to the picture, these ratios are:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}$$

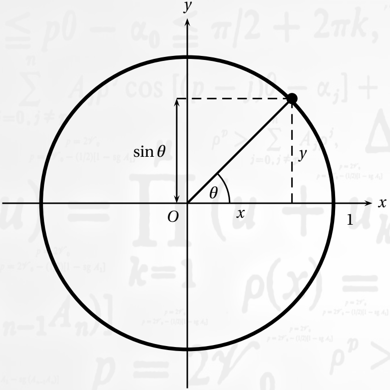
Positive and negative angles

Consider the Cartesian plane *Oxy* as shown below. The coordinate axes divide the plane into four quadrants.



Positive angles are measured anti-clockwise from the *x*-axis. Negative angles are measured clockwise from the *x*-axis. So the angle −45° is equivalent to the angle 315° (found by adding 360°).

The sine function $f(\theta) = \sin\theta$



The picture above shows a a unit circle (radius 1) and a right-angled triangle.

The function $\sin\theta$ can be defined on this unit circle.
The sine ratio is defined as

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

but in this case the length of the hypotenuse is 1 unit.

$$\sin\theta = y$$

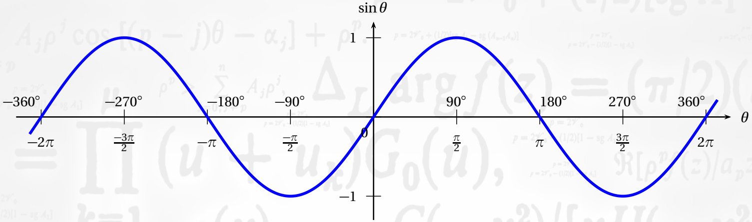
The value of $\sin\theta$ is equal to the length of the side opposite the angle θ as shown in the picture.

Interactive: `http://www.geogebratube.org/student/m43021`.

The graph of $\sin\theta$

Each angle θ corresponds to a value for y (which is $\sin\theta$).

By graphing θ against $\sin\theta$ we get this curve:



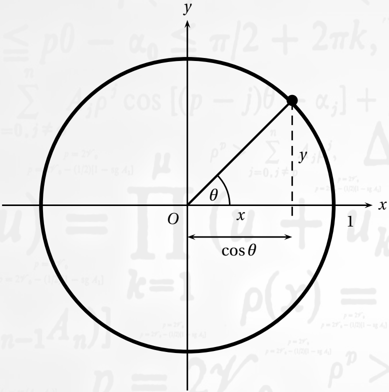
Negative angles are obtained by moving the point *clockwise* around the unit circle (so −45° is the same value as 315° etc).

The scale in degrees is shown above the θ-axis. Equivalent angles in radians, given in terms of π, are shown below the x-axis.

The following properties should also be noted:

- The period of the function is 360° (or 2π radians)
- Values of sine are found in the interval $-1 \le \sin\theta \le 1$.

The cosine function $f(\theta) = \cos\theta$



The picture above shows a a unit circle (radius 1) and a right-angled triangle.

The function $\cos\theta$ can be defined on this unit circle.
The cosine ratio is defined as

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

but in this case the length of the hypotenuse is 1 unit.

$$\cos\theta = x$$

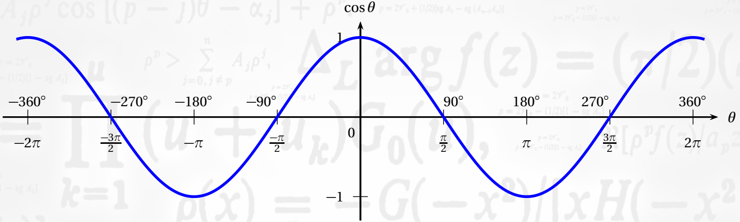
The value of $\cos\theta$ is equal to the length of the side adjacent the angle θ as shown in the picture.

Interactive: `http://www.geogebratube.org/student/m43021`.

The graph of $\cos\theta$

Each angle θ corresponds to a value for x (which is $\cos\theta$).

By graphing θ against $\cos\theta$ we get this curve:

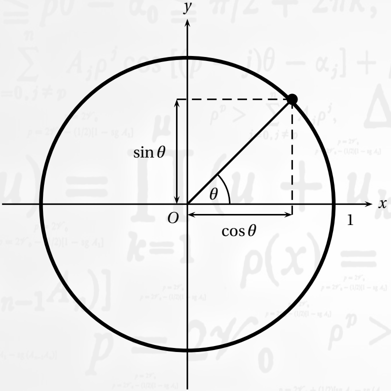


Negative angles are obtained by moving the point *clockwise* around the unit circle (so −90° is the same value as 270° etc).

The following properties should also be noted:

- The period of the function is 360° (or 2π radians)
- Values of cosine are found in the interval $-1 \leq \cos\theta \leq 1$.

The tangent function $f(\theta) = \tan\theta$



The tangent ratio is defined as

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

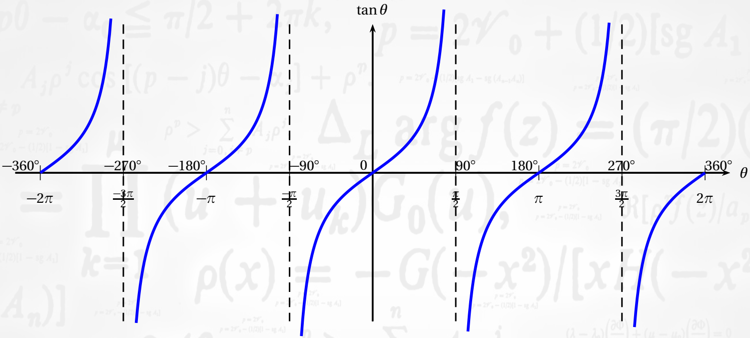
The unit circles shown previously show that this ratio is equivalent to

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Interactive: `http://www.geogebratube.org/student/m43021`.

The graph of $\tan\theta$

A graph of values of θ against $\tan\theta$ looks like this:

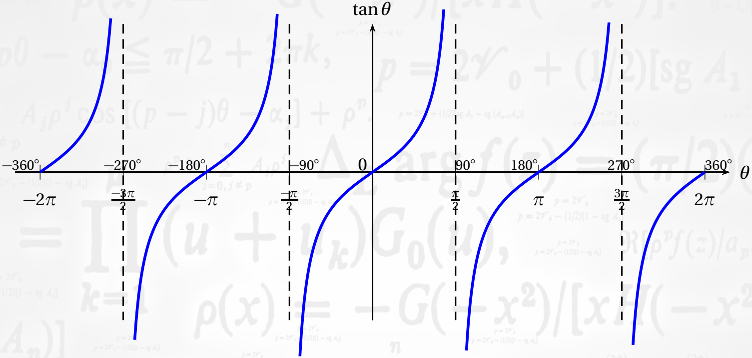


Negative angles are obtained by moving the point *clockwise* around the unit circle (so $-90°$ is the same value as $270°$ etc).

The following properties should also be noted:

- The period of the function is $180°$ (or π radians)
- Values of tangent can be any real number (the interval $-\infty < \tan\theta < \infty$).

Some more features to note about this graph.

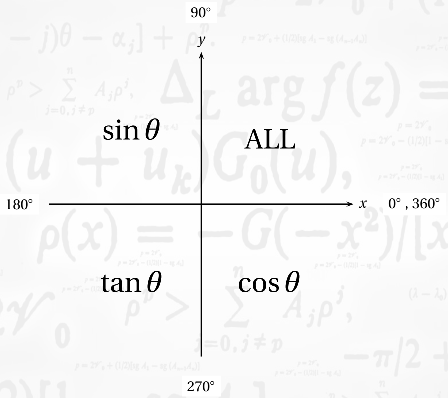


The graph of $y = \tan\theta$ has discontinuities (breaks) at places where $\cos\theta = 0$ ($\pm 90°, \pm 270°$, etc). The tangent function is not defined at these values.

The dashed lines on the graph at these values are called *vertical asymptotes* and the curve approaches but never touches these lines.

The four quadrants

With reference to the graphs of sine, cosine and tangent, the signs of those functions in each quadrant is shown below.



Consideration of the quadrant is important because the angle returned by a calculator or computer is the principal value and may not correspond to the actual angle required.

Trig ratios of common angles

$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$
θ rad	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—

Trigonometric ratios of some common angles.

Many modern calculators will automatically return these values.

Ratios for other angles can be obtained by thinking about the properties of each trig function.

For example, the graph of $\sin\theta$ is symmetric about the line $\theta = 90°$ so that

$$\sin 150° = \sin 30° = \frac{1}{2}$$

Test yourself

You should be able to solve the following problems if you have understood everything in these notes.

1. Sketch the graph of $y = \sin x$ and use it to estimate the value of $\sin(-20°)$.
2. Sketch the graph of $y = \cos x$ and use it to estimate the value of $\cos 300°$.
3. Express $\sin(-253°)$ as $\sin k$ where $0 \leq k < 360°$
4. Express $\cos 1215°$ as $\cos k$ where $0 \leq k < 360°$
5. Find two values of x which satisfy $\tan x = 1$ in $0 \leq x < 360°$

Answers:

1. $\sin(-20°) \approx -0.3$
2. $\cos(300°) = 0.5$
3. $\sin(-253°) = \sin 107°$
4. $\cos 1215° = \cos 135°$
5. $x = 45°, 225°$.