Sigma Notation

Series 1

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Introduction to Series

A series is the *sum* of the terms in a sequence.

A finite series stops after a finite number of terms.

An infinite series has no final term.

For example, the series

$$2 + 4 + 6 + \ldots + 40$$

is a finite series with 20 terms. The dots indicate the terms follow the same pattern as the previous terms.

The series

$$2 + 4 + 8 + 16 + \ldots$$

is an infinite series. In this case the dots indicate that the pattern continues in the same way and that it continues forever.

The rules for generating the terms in a series will be discussed in later presentations.

Sigma notation

Consider the series given by

$$1+2+4+8+16+32+64+128+256$$

Each term is a power of 2 and so we can write this series as:

$$2^0+2^1+2^2+2^3+2^4+2^5+2^6+2^7+2^8$$

We can use Sigma (Σ) notation to write out series in a much more compact way:

$$\sum_{n=0}^{8} 2^n$$

The index n takes all integer values between $n=0$ (the lower limit) and $n=8$ (the upper limit) in consecutive terms.

The Σ symbol tells us to add these terms.

The Greek letter Σ corresponds to the western letter 'S', which stands for *summa* (meaning summation)

Evaluating a sum

Evaluate $\sum_{n=2}^{7} n^3$

We can write out the series in full:

$$\begin{aligned} \sum_{n=2}^{7} n^3 &= 2^3+3^3+4^3+5^3+6^3+7^3 \\ &= 8+27+64+125+216+343 \\ \therefore \sum_{n=2}^{7} n^3 &= 783 \end{aligned}$$

In this example there were 6 terms in the series.

Given the summation $\sum_{a}^{b}$, the number of terms in the series is $b-a+1$.

Evaluating series by writing them out in full and then summing them is very inefficient! We will study two important classes of series (arithmetic and geometric) and investigate methods for evaluating sums more efficiently.

Conversion to Sigma notation

Write out the following series in Σ-notation:

$$2+4+6+8+10+\ldots+30$$

Each new term is obtained by multiplying the term number by 2

The 2nd term is $2\times 2=4$. The 5th term is $2\times 5=10$....and so on.

The final term is 30 so this must be the 15th term (since $2\times 15=30$). This means we can write the series in this equivalent form:

$$2(1)+2(2)+2(3)+2(4)+2(15)+\ldots+2(15)$$

This series is represented by Sigma notation as $\sum_{n=1}^{15} 2n$.

Conversion to Sigma notation

Give the following series in Σ-notation:

$$6+10+14+18+22+\ldots+62$$

This series, where the difference between each term is 4, is related to the series:

$$4+8+12+16+20+\ldots+60$$

That series is represented by Sigma notation as $\sum_{n=1}^{15} 4n$.

Each term in the given series is 2 more than this series.

Therefore it should be written: $\sum_{n=1}^{15}(4n+2)$.

Test yourself...

You should be able to solve the following problems if you have understood everything in these notes.

1. Evaluate $\sum_{n=3}^{5} \frac{3}{n}$
2. How many terms are in the series $\sum_{n=2}^{40} (2n+1)$?
3. Express the infinite series $5+2-1-4-\ldots$ using Sigma notation.
4. Express $0-1+4-9+16-\ldots+100$ as a Sigma sum.

Answers:

1. $\sum_{n=3}^{5} \frac{3}{n} = \frac{3}{3}+\frac{3}{4}+\frac{3}{5} = \frac{47}{20}$
2. 39 terms
3. $\sum_{n=0}^{\infty} (5-3n)$
4. $\sum_{n=0}^{10} (-1)^n \times n^2$.
   Here, $(-1)^n$ alternates the sign between $\pm$.