Pairs of lines

Geometry 1

INU0114/514 (Maths 1)

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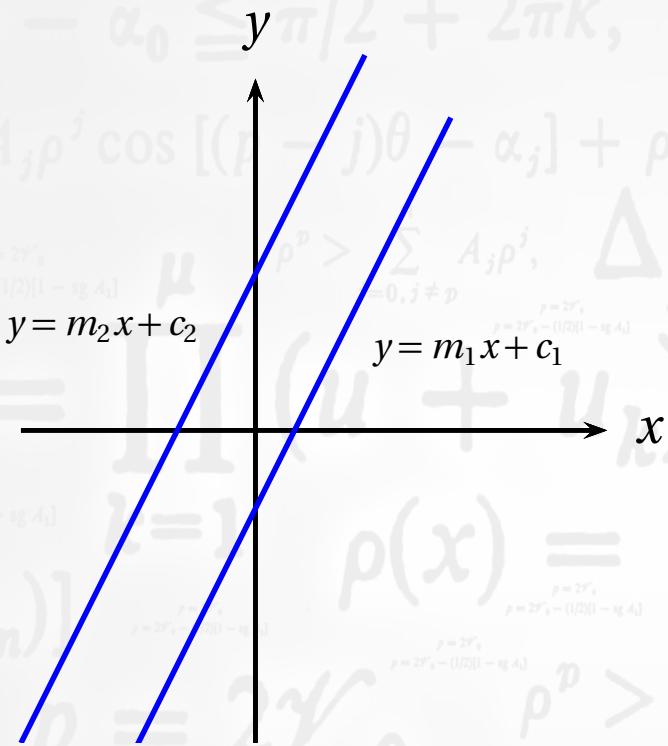
Objectives

The notes in this presentation are about properties of pairs of lines in two dimensions:

- Parallel lines
- Finding points of intersection between non parallel lines.
- Perpendicular lines
- Tangent lines and normal lines

Parallel straight lines



Given two straight lines

$$y = m_1x + c_1$$
$$y = m_2x + c_2$$

Then the lines are *parallel* if and only if their gradients are the same:

$$m_1 = m_2$$

The values of c_1 and c_2 are different because the lines cross the y axis at different places.

Two lines with the same slope will never intersect.

Parallel lines

Given the line $y = 2x - 3$, find the the equation of a parallel line passing through the point $(4, -1)$.

The gradient of the given line is $m_1 = 2$.

The gradient of a parallel line is the same; so $m_2 = 2$.

Using the general equation of a straight line we must have

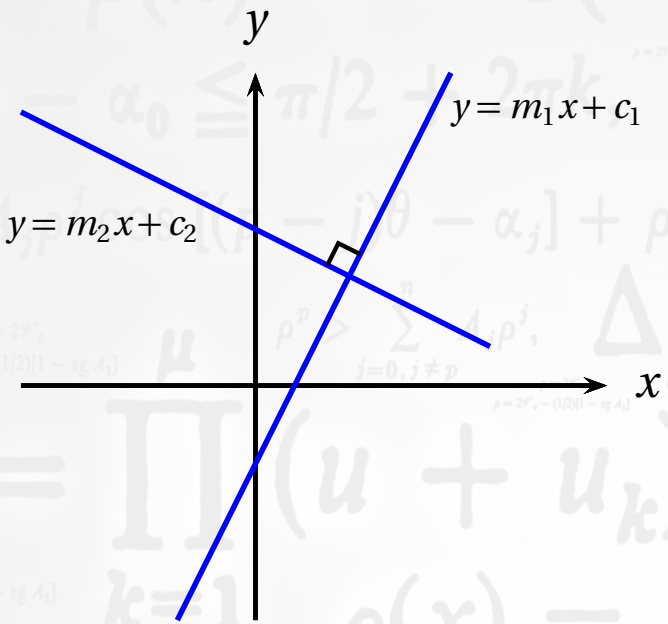
$$y = 2x + c$$

And since it passes through $(4, -1)$ then

$$-1 = 2(4) + c \qquad \Rightarrow c = -9$$

The parallel line is given by $y = 2x - 9$.

Perpendicular straight lines



For two lines given by

$$y = m_1x + c_1$$
$$y = m_2x + c_2$$

They are *perpendicular* if they intersect with an angle of 90°.

For perpendicular lines then the following relationship will also be true:

$$\boxed{m_1m_2 = -1}$$

e.g. given the line $y = 3x - 5$ the gradient of a perpendicular line is $-\frac{1}{3}$.

To find the *equation* of a perpendicular line we would also need to know a point on that line.

Perpendicular lines

Given the line $y = 10 - 3x$, find the the equation of a perpendicular line passing through the point $(6, 1)$.

The gradient of the given line is $m_1 = -3$.

The gradient of a perpendicular line is found from $m_1 m_2 = -1$.

Therefore $m_2 = \frac{1}{3}$.

Using the general equation of a straight line we must have

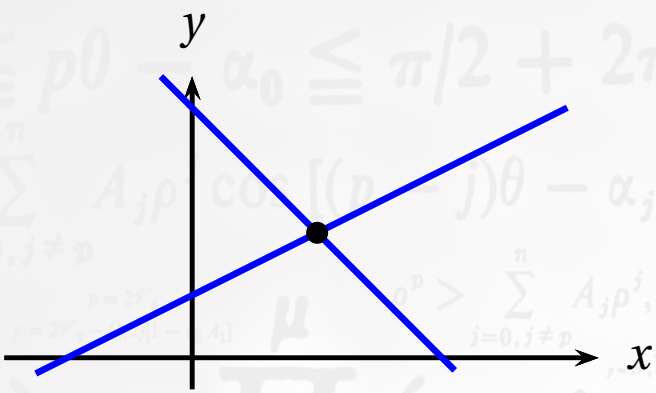
$$y = \tfrac{1}{3}x + c$$

And since it passes through $(6, 1)$ then

$$1 = \tfrac{1}{3}(6) + c \qquad \Rightarrow c = -1$$

The perpendicular line is $y = \frac{1}{3}x - 1$.

Intersection points



If two straight lines have different gradients then they will intersect at some point.

The point of intersection can be calculated by equating the line equations and solving for x.

Intersection of two lines

Find the intersection of the lines

$$y = 4 - x \quad \text{and} \quad y = \tfrac{1}{2}x + 1$$

At the intersection points, the y-value is shared so we equate the expressions:

$$4 - x = \tfrac{1}{2}x + 1$$

Rearrange and solve to get $x = 2$.

Substitute into either of the original equations to get $y = 2$.

The intersection is at the point $(2, 2)$.

Test yourself...

Try to answer the following questions on this topic:

1. Two lines have gradients $-\frac{2}{7}$ and $\frac{7}{2}$ respectively. Are they perpendicular?
2. Find the equation of a parallel line to $y=-3x+10$ which passes through $(1,5)$
3. Find the equation of a perpendicular line to $y=\frac{1}{4}x-2$ which passes through $(-2,8)$
4. Two lines: $y=2x+5$ and $y=-2x+11$. Find the intersection point.

Answers:

1. Yes: $-\frac{2}{7}\times(\frac{7}{2})=-1$
2. $y=-3x+8$
3. $y=-4x$
4. $(\frac{3}{2},8)$