

SIMULTANEOUS EQUATIONS

ALGEBRA 1

INU0114/514 (MATHS 1)

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Overview

In this presentation we will review how to solve simultaneous equations.

- Solve by the method of elimination
- Solve by the method of substitution.

We will limit the study to linear equations for the time being (equations not containing powers, roots or other functions).

Introduction

Simultaneous equations

This is the name given when we have to solve **two equations** with **two unknowns**. We must solve the equations at the same time to find the solutions.

For example the equations

$$x + y = 10$$

$$2x - y = 11$$

can be solved to give $x = 7$ and $y = 3$ as the solution.

(Verify by substitution that they are solutions).

Solution by elimination

Solve the equations

$$3x + y = 14$$

$$2x - y = 11$$

To solve we notice that $+y$ and $-y$ are present in the equations.

Adding corresponding sides of the equations

$$\begin{array}{r} 3x + y = 14 \\ + 2x - y = 11 \\ \hline 5x = 25 \end{array}$$

eliminates the y to leave a single equation. Therefore $x = 5$

To find the other solution we substitute x into one of the given equations.

E.g. using the first:

$$3(5) + y = 14$$

$$15 + y = 14 \quad \therefore y = -1$$

Solution by elimination (harder)

Solve the equations

$$2x - y = -10$$

$$3x + 5y = 11$$

There are no obvious cancellations this time. But we can manipulate the equations so that elimination is possible.

Multiply the first equation by 3 and the second by -2 and we get:

$$6x - 3y = -30$$

$$-6x - 10y = -22$$

Adding the new equations eliminates x to give $-13y = -52$

Therefore $y = 4$

Substitute y into any of the given equations to find x .

E.g. using the first:

$$2x - 4 = -10$$

$$2x = -6 \quad \therefore x = -3$$

Solution by substitution

Solve the equations

$$\begin{aligned}x - y &= -13 \\5x - 2y &= -56\end{aligned}$$

Rearrange the first equation to get $x = y - 13$

Now substitute this into the second equation:

$$5(y - 13) - 2y = -56$$

Simplify and solve:

$$\begin{aligned}5y - 65 - 2y &= -56 \\3y &= 9 \\y &= 3\end{aligned}$$

To find the other solution we substitute y into one of the given equations.

E.g. using the first

$$x - 3 = -13 \quad \therefore x = -10$$

Solution by substitution (harder)

Solve the equations

$$15x + 25y = -14$$

$$10x - 5y = 8$$

Rearrange the second equation to get $y = 2x - \frac{8}{5}$

Now substitute this into the first equation:

$$15x + 25\left(2x - \frac{8}{5}\right) = -14$$

Simplify and solve:

$$15x + 50x - 40 = -14$$

$$65x = 26 \quad \therefore x = \frac{2}{5}$$

To find the other solution we substitute y into one of the previous equations.

E.g. using $y = 2x - \frac{8}{5}$

$$y = 2\left(\frac{2}{5}\right) - \frac{8}{5} \quad \therefore y = -\frac{4}{5}$$

Word problems

Let's see some examples where we must extract the equations from a description of a problem.

Simultaneous equations

Two numbers have a sum of 496 and the difference between them is 204. What are the two numbers?

We can make two equations from this description. Let the two numbers be a and b :

$$a + b = 496$$

$$a - b = 204$$

We can solve using the usual methods; in this case add the equations to get

$$2a = 700 \quad \therefore a = 350$$

Then use the first equation: $b = 496 - a = 496 - 350 = 146$.

The two numbers are 146 and 350.

Another word problem

15 litres of 16% disinfectant solution is to be made from 20% and 14% solutions. How much of those solutions should be used?

Let x be the number of litres of 20% solution. Let y be the number of litres of 14% solution.

We're trying to make 15 litres of 16% solution; that means the total volume of 15 litres is constant:

$$x + y = 15 \quad (1)$$

The total amount of disinfectant is $15 \times 0.16 = 2.4$ litres. In equation form:

$$0.2x + 0.14y = 2.4 \quad (2)$$

We have the two equations we need; we should be able to solve them using any of the methods seen so far. Multiply equation (1) by 5 to make it easier to eliminate:

$$x + 0.7y = 12 \quad (3)$$

Subtract equation (3) from equation (1) to get $0.3y = 3$ and solve to get $y = 10$. Substitute into equation (1) to find $x = 5$.

We need 5 litres of 20% and 10 litres of 16% solution.

Test yourself...

You should be able to solve the following simultaneous equations.

- ① $x + y = 11$ and $5x - y = 19$.
- ② $3x - 3y = 12$ and $4y - x = -10$.
- ③ $3x + 10y = 1.93$ and $-4x + 5y = -0.19$

Answers:

- ① $x = 5, y = 6$
- ② $x = 2, y = -2$
- ③ $x = 0.21, y = 0.13$